A Multiscale FEM for 2D Photonic Crystal Bands

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Abstract

A Multiscale Finite Element Method (MSFEM) for wave propagation in locally periodic media, e.g., Photonic Crystals (PhC), will be presented [1]. We consider wave propagation at wavelengths of the size of the local periodicity. In this case homogenisation techniques are not applicable. The MSFEM uses two-scale basis functions inside the PhC. As micro functions we use Bloch modes, which are computed as FEM solutions for a fully periodic PhC [7]. The macro functions are piecewise polynomials of degree p^{mac} which are supported over many periods of the crystal. We will numerically show that such a multiscale basis is very efficient as only a constant number of these functions are needed to simulate arbitrary large, finite PhCs with a constant L_2-error. In contrast, for standard discretisation schemes like FD, PWM, h- or p-FEM more and more basis functions are required when the number of scatterers increases inside the computational domain. We will explain how to use this multiscale basis to construct a conforming FEM which is coupled to a discretisation of the exterior domain. In particular, we will show how to numerically integrate the highly oscillatory two-scale functions with constant computational effort. We will verify the properties of the MSFEM by numerical experiments for PhC bands (see Fig. 1), an infinite band of locally-periodic dielectric scatterers in 2D.

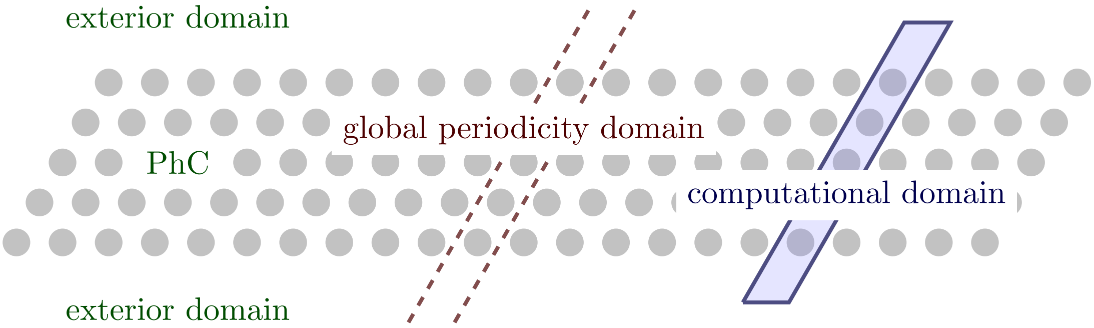

Figure 1: An example Photonic Crystal band

References

[1] Brandsmeier, H., Schmidt, K., and Schwab, C. A Multiscale FEM for 2D Photonic Crystal Bands. SAM Report, in preparation.

[2] Hou, T., and Wu, X. A multiscale finite element method for elliptic problems in composite materials and porous media. *Journal of Computational Physics 134*, 1 (1997), 169–189.

[3] Matache, A. Sparse Two-Scale FEM for Homogenization Problems. *J. Sci. Comput. 17*, 1 (2002), 659–669.

[4] Matache, A., Babuška, I., and Schwab, C. Generalized p-FEM in homogenization. *Numer. Math. 86*, 2 (2000), 319–375.

[5] Melenk, J., and Babuška, I. The partition of unity finite element method: basic theory and applications. *Computer methods in applied mechanics and engineering 139*, 1-4 (1996), 289–314.

[6] Rüegg, A. W., Schneebeli, A., and Lauper, R. Generalized hp-FEM for lattice structures. Research report no. 2002-23, ETH Zürich, Seminar of Applied Mathematics, 2002.

[7] Schmidt, K., and Kauf, P. Computation of the band structure of two-dimensional Photonic Crystals with *hp* Finite Elements. *Comput. Methods Appl. Mech. Engrg. 198* (2009), 1249–1259.