Nonlinear spectral problems with applications to photonic crystals

Christian Engström[1],

[1]Seminar for Applied Mathematics, ETH Zurich, Rämistrasse 101, Zurich 8092, Switzerland,
christian.engstroem@sam.math.ethz.ch
Electromagnetic Fields and Microwave Electronics Laboratory,
ETH Zurich, Gloriastrasse 35, Zurich 8092, Switzerland

1 Introduction

Dielectric and metallic photonic crystals are promising materials for controlling and manipulating electromagnetic waves [1]. For frequency independent material models considerable mathematical progress has been made [2]. In the frequency dependent case, however, the nonlinearity of the spectral problem complicates the analysis. We study the spectrum of a scalar operator-valued function with periodic coefficients, which after application of the Floquet transform become a family of spectral problems on the torus. The frequency dependence of the material parameters lead to spectral analysis of a family of holomorphic operator-valued functions.

2 Results

We show that the spectrum for a passive material model consists of isolated eigenvalues of finite geometrical multiplicity. These eigenvalues depend continuously on the quasi momentum and all non-zero eigenvalues have a non-zero imaginary part whenever losses (absorption) occur [3].

Lorentz permittivity model, which is a common model for solid materials, lead to a rational eigenvalue problem. We study both the self-adjoint case and the non-self-adjoint case. Moreover, a high-order discontinuous Galerkin method is used to discretize the operator-valued function, and the resulting matrix problem is transformed into a linear eigenvalue problem [4]. Finally, we use an implicitly restarted Arnoldi method to compute approximate eigenpairs of the sparse matrix problem.

References

[1] K. Sakoda, Optical properties of photonic crystals, Springer-Verlag, Heidelberg, 2001.
[2] P. Kuchment, Floquet theory for partial differential equations, Birkhäuser, Basel, 1993.
[3] C. Engström, *On the spectrum of a holomorphic operator-valued function with applications to absorptive photonic crystals*, To appear in Mathematical Models and Methods in Applied Sciences, 2010.
[4] C. Engström and M. Wang, *Complex dispersion relation calculations with the symetric interior penalty method*, To appear in International Journal for Numerical Methods in Engineering, 2010.