Analysis and numerical analysis of a nonlinear Stokes problem arising in glaciology

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In this talk, a three dimensional problem describing the motion of glaciers is addressed. An ice flow model that accounts for shearing and sliding is coupled with a complex mass balance model accounting for climate. This allows a numerical simulation of Aletsch glacier over the last 120 years to be performed, see the figure. Simulations over the coming century can be investigated according to several climatic scenarios [1].

At each time step, the velocity of ice is obtained by solving a stationary non-linear Stokes problem with mixed Dirichlet and non-linear Robin condition along the bedrock-ice interface. The well-posedness of the weak formulation is proved [2]. A finite element discretization based on stable inf-sup spaces for velocity and pressure is proposed and *a priori* error estimates are established by using a quasi-norm technique [3]. Several schemes are proposed to solve the non-linearity and are proved to be convergent.

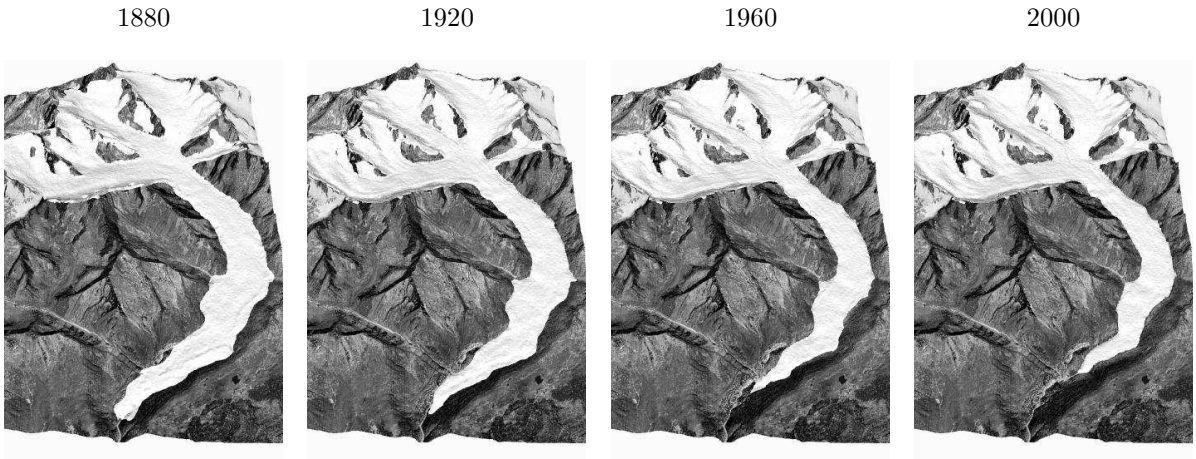

Figure 1: Simulation of Aletsch glacier over the period 1880-2000.

References

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