MODEL ORDER REDUCTION BY REDUCED BASIS METHODS FOR FLOW SIMULATION AND SHAPE OPTIMIZATION IN HAEMODYNAMICS

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Computational fluid dynamics provides models and methods to describe, simulate and control blood flows in cardiovascular districts, aiming at the analysis of blood circulation and prevention of cardiovascular diseases. Hence, a detailed understanding of local haemodynamics phenomena and the effect of vascular wall modication on flow patterns can have useful clinical applications especially in surgical procedures. The evaluation of wall shear stresses in a stenosed vessel or the minimization of vorticity in a cardio-vascular bypass anastomosis [**?**, **?**] are just two examples of problems in which we are interested considering flows which are highly dependent on vessels configuration combined with shape optimization of graftings.
These problems usually imply big computational efforts, basically due to fluid dynamics phenomena and geometrical complexity. Moreover, we need to face with many query problems, due to multiple evaluations of outputs depending on field variables during an optimization procedure, or to repetitive simulations on different geometries in a patient-dependent framework in view of a real-time solution. For these reasons, looking for computational efficiency in numerical methods and algorithms is mandatory, making the interplay between scientific computing and new reduction strategies a crucial topic. For an efficient model order reduction, reduced basis (RB) methods [**?**, **?**], built upon a high-fidelity "truth" finite element (FE) approximation, combined with free-form deformations (FFD) techniques for efficient shape parametrization are introduced [**?**, **?**], decreasing both the computational effort and the geometrical complexity. A parametrized RB approach can provide rapid and reliable results in real-time and many-query contexts; reliability is ensured by rigorous a posteriori error bounds, while rapid response is ensured by a suitable Offline–Online computational strategy [**?**]. Free-form deformations are built upon a low-dimensional parametrization and enable global shape deformations by acting on a small set of control parameters, resulting in a flexible, versatile and accurate tool [**?**]. Some results for haemodynamics applications are presented, dealing with shape optimization of parametrized configurations and real-time simulation of complex flows, discussing computational advantages and efficiency, obtained by geometrical and computational model order reduction.

References

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