A stabilized finite volume element method for the Stokes' problem

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Abstract

We will devote ourselves to the study of a particular stabilized finite volume element (FVE) method constructed on the basis of a conforming finite element formulation for the Stokes equations, where the velocity and pressure fields are approximated by linear piecewise polynomials. Among the wide class of stabilized finite element formulations available from the literature, such as Streamline-Upwind/Petrov-Galerkin (SUPG), Galerkin-Least-Squares (GLS) and other methods, in this contribution we include a stabilization technique similar to the one introduced by Araya *et al.* [**?**], in which a Petrov-Galerkin approach is used to enrich the trial space with bubble functions being solutions to a local problem involving the residual of the momentum equation, which can be solved analytically. By enriching the velocity space using a *multiscale* approach combined with static condensation, the resulting FE method includes the classical GLS additional terms at the element level and a suitable jump term on the normal derivative of the velocity field at the element boundaries. Then, the essential point is to appropriately connect the finite element and FVE formulations. We also carry out a superconvergence analysis of the approximate solution. The main goal is to improve the current accuracy of the approximation by applying a postprocessing technique constructed on the basis of a projection method presented in [**?**, **?**]. The technique consists in projecting the FVE space to another approximation space (possibly of higher order) related to a coarser mesh. Finally, some numerical experiments that confirm the predicted behavior of the method are provided.

References

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