Conservative FEM-simulation of the Einstein-Dirac system in spherically symmetric spacetime

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Abstract

Our work considers the massive spherically symmetric Einstein–Dirac system obtained from the relativistic two–fermion singlet state. First, we reviewed classical construction of the Einstein–Dirac equations in a general globally hyperbolic spacetime, with additional help of recent results by A. N. Bernal and M. Sànchez about smoothness of spacelike slicing. Some new analytical results have been found for the spherically symmetric case, e.g. an estimate of total ADM–mass against spatial Sobolev norms of the Dirac fields. In particular, we introduced a probably well posed initial value formulation for massive spherically symmetric Einstein–Dirac system in Schwarzschild coordinates. Further, we developed a spatial Galerkin method, suitable for finding numerical solutions of this initial value problem. The discretization scheme features exact conservation of the total electric charge, and allows for spatial mesh adaption based on physical arclength. We also derived error estimates and conditions that imply L^2–convergence of numerical solution to exact solution along lattice refinements. Based on these algorithms we realized a new platform independent and highly optimized UNIX commandline simulation software implemented in C++. In particular, our software allows for computations of static configurations as well as time evolutions obtained from suitable initial configurations w.r.t. the same discretization scheme and data structures. Numerical experiments and convergence tests due to Richardson's extrapolation confirm excellent robustness and convergence properties of our approach. Finally, we applied our software in order to confirm and significantly extend recent results about critical behaviour near the black hole threshold as well as structure and stability of static solutions.

References

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